Co-Scheduling Algorithms for High-Throughput Workload Execution

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Scheduling in Aussois 2014

Applications

p identical processors
n moldable applications, or tasks, T_i, s.t.,

- p_i is the minimal number of processors needed for T_i;
- $t_{i,j}$ is the execution time of task T_i with j processors;
- $work(i,j) = j \times t_{i,j}$ is the corresponding work.

We assume the following for $1 \leq i \leq n$ and $p_i \leq j < p$:

Non increasing execution time: $t_{i,j+1} \leq t_{i,j}$
Non decreasing work: $work(i, j+1) \geq work(i,j)$

Co-schedules

A co-schedule partitions the n tasks into groups (called *packs*), so that (i) all tasks from a given pack start their execution at the same time; and (ii) two tasks from different packs have disjoint execution intervals.

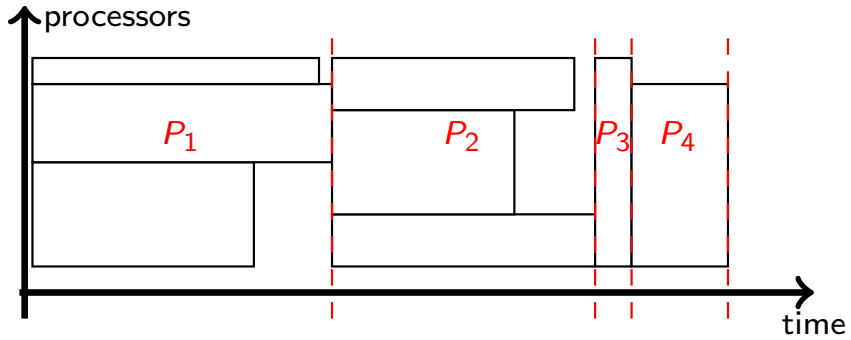

Figure : A co-schedule with four packs P_1 to P_4.

Definition (k-IN-p-CoSchedule optimization problem)

Given a fixed constant $k \leq p$, find a co-schedule with at most k tasks per pack that minimizes the execution time.

The most general problem is when $k = p$, but in some frameworks we may have an upper bound $k < p$ on the maximum number of tasks within each pack.

Related work

Performance bounds for level-oriented two-dimensional packing algorithms, Coffman, Garey, Johnson:
Strip-packing problem, parallel tasks (fixed number of processors), approx-algo based on "shelves".

Scheduling parallel tasks: Approximation algorithms, Dutot, Mounié, Trystram:
Use this model to approximate the moldable model; they studied the p-IN-p-CoSchedule for identical moldable tasks (poly with DP).

Widely studied for sequential tasks.

Polynomial instances

Theorem

The 1-IN-*p*-CoSchedule *and* 2-IN-*p*-CoSchedule *problems can both be solved in polynomial time.*

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The 1-IN-p-COSCHEDULE *and* 2-IN-p-COSCHEDULE *problems can both be solved in polynomial time.*

Proof.

If there is a pack with exactly task T_i and T_j, then its execution time is $\min_{j=1..p} \left(\max(t_{i,p-j}, t_{i',j})\right)$.

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We then construct the complete weighted graph $G = (V, E)$, where $|V| = n$, and

$$e_{i,j} = \begin{cases} t_{i,p} & \text{if } i = j \\ \min_{j=1..p}\left(\max(t_{i,p-j}, t_{i',j})\right) & \text{otherwise} \end{cases}$$

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Finally, solving MIN-WEIGHT-PERFECT-MATCHING on G gives the best solution for 2-IN-p-COSCHEDULE. □

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The greedy algorithm that, while there remains available processors, assigns one to the largest task (with their current processor assignement), returns an optimal solution. We call this procedure Optimal-1-pack-schedule

Intractability

Theorem

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We reduce this problem to 3-PARTITION: Given an integer B and $3n$ integers $a_1, \ldots, a_{3n}$, can we partition the $3n$ integers into n triplets, each of sum B? This problem is strongly $\mathcal{NP}$-hard so we can encode the a_i's and B in unary.

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We build the following instance $\mathcal{I}_2$ of 3-IN-p-CoSchedule:

- $p = B$ processors
- a deadline $D = n$
- $3n$ tasks T_i such that $t_{i,j} = 1 + \frac{1}{a_i}$ if $j < a_i$, $t_{i,j} = 1$ otherwise.

(See that the $t_{i,j}$'s verify the constraints on the work and execution time.)

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Then necessarily any solution of $\mathcal{I}_2$ has n packs each of size 1 with exactly 3 tasks in it, and the sum of the weight of those tasks sums up to B.

□

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We reduce those problems to the same instance of the 3-IN-p-CoSchedule problem, to which we further add:

- $n(k-3)$ buffer tasks such that $t_{i,j} = \max\left(\frac{B+1}{j}, 1\right)$;
- the number of processors is now $p = B + (k-3)(B+1)$;
- the deadline remains $D = n$.

Again, we need to execute each pack in unit time and at most n packs. The only mean to do so is to execute within each pack $k-3$ buffer tasks on $B+1$ processors.

Open problems?

k-IN-p-CoSchedule for $k \geq 3$ is $\mathcal{NP}$-complete, 2-IN-p-CoSchedule $\in \mathcal{P}$. What about 3-IN-3-CoSchedule?

Some ideas:

- Obvious solution to "exactly-3-IN-3-CoSchedule", however "exactly-3-IN-p-CoSchedule" is $\mathcal{NP}$-complete
- DP to solve the 3-IN-3-CoSchedule problem where a batch can have 1 or 3 elements (but not two).

Chances are, it is solvable in polynomial time.

Optimal solution

Theorem

The following integer linear program characterizes the k-IN-p-CoSchedule problem, where the unknown variables are the $x_{i,j,b}$'s (Boolean variables) and the y_b's (rational variables), for $1 \leq i, b \leq n$ and $1 \leq j \leq p$:

$$
\begin{array}{lll}
\text{Minimize} \sum_{b=1}^{n} y_b & \text{subject to} & \\
(i)\ \sum_{j,b} x_{i,j,b} = 1, & 1 \leq i \leq n & \\
(ii)\ \sum_{i,j} x_{i,j,b} \leq k, & 1 \leq b \leq n & (1) \\
(iii)\ \sum_{i,j} j \cdot x_{i,j,b} \leq p, & 1 \leq b \leq n & \\
(iv)\ x_{i,j,b} \cdot t_{i,j} \leq y_b, & 1 \leq i, b \leq n, 1 \leq j \leq p &
\end{array}
$$

Approximation Results

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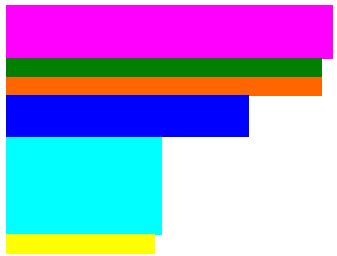

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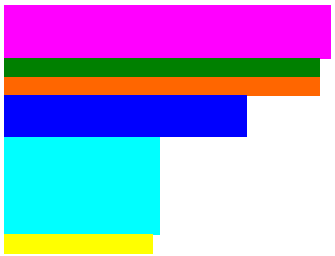

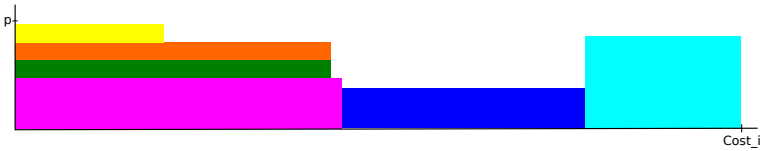

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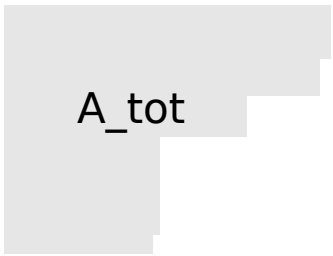

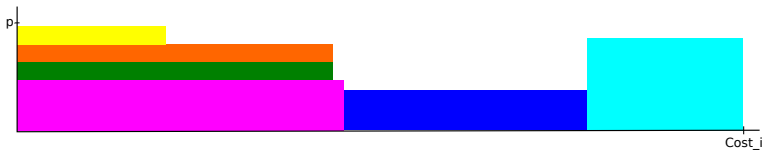

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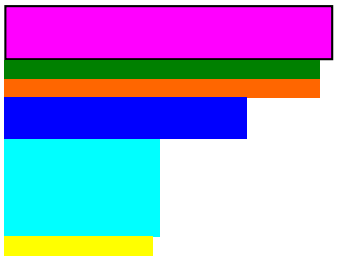

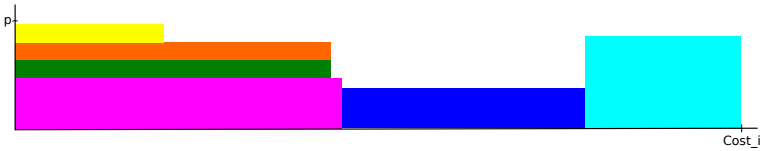

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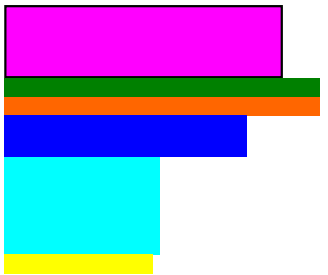

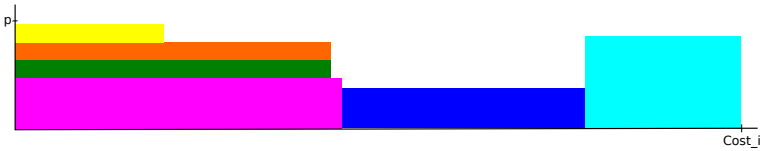

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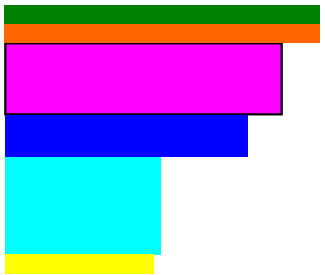

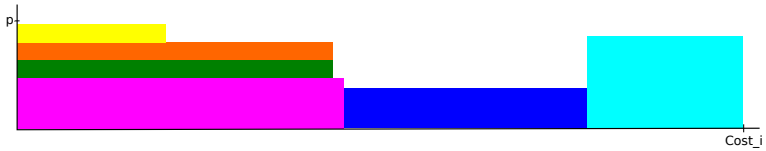

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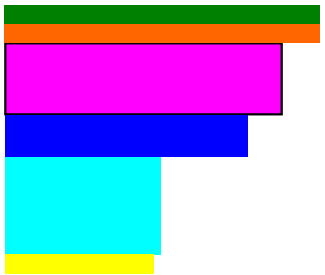

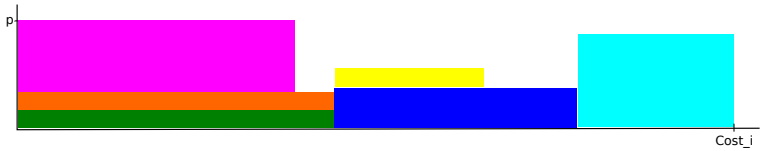

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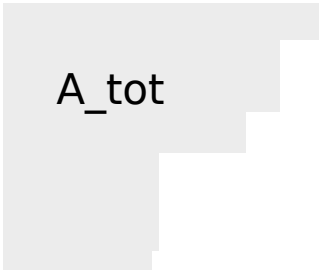

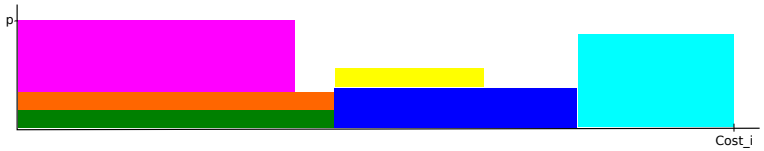

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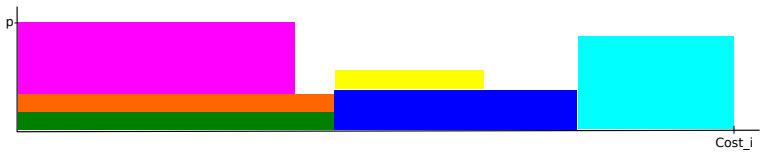
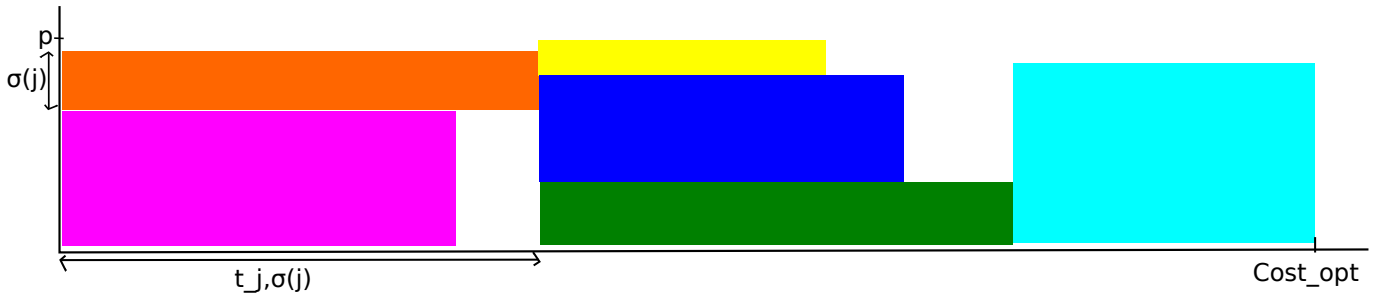


Analysis

A couple of important points:

- Given an optimal solution OPT , $\forall j, t_{j,\sigma^{(\mathrm{OPT})}(j)} \leq \mathrm{COST}_{\mathrm{OPT}}$ and $A_{\mathrm{OPT}} \leq p\mathrm{COST}_{\mathrm{OPT}}$.



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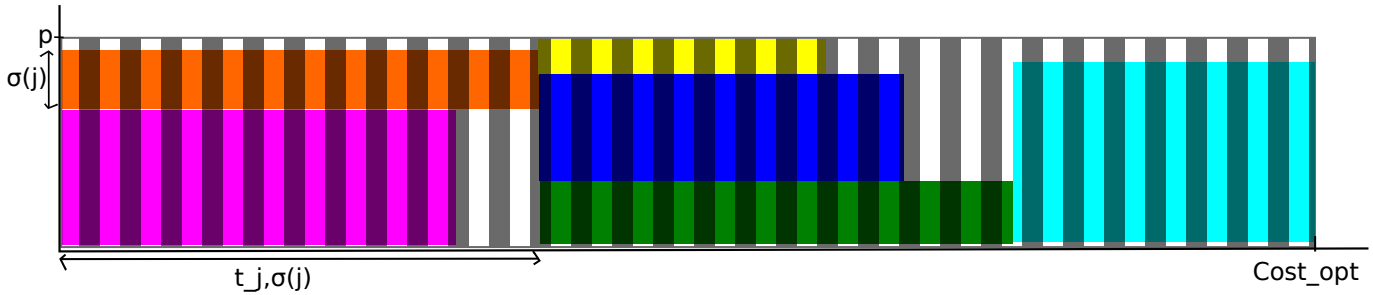

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  $\sigma^{(i)}(j) = p_j$ or $t_{j,\sigma^{(i)}(j)-1} \geq t_{\mathrm{max}}(i) > \mathrm{COST}_{\mathrm{OPT}} \geq t_{j,\sigma^{(\mathrm{OPT})}(j)}$

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$$\begin{aligned}\mathrm{COST}_{i_0} &\leq 3\max\left(t_{\max}(i_0), \frac{A_{\mathrm{tot}}(i_0)}{p}\right)\\ &\leq 3\max\left(\mathrm{COST}_{\mathrm{OPT}}, \frac{A_{\mathrm{OPT}}}{p}\right) \leq 3\mathrm{COST}_{\mathrm{OPT}}\end{aligned}$$

Heuristics

In all heuristics (even randoms), once the different packs are chosen, we always run Optimal-1-pack-schedule on each pack.

Random-Pack: generates the packs randomly: randomly chooses an integer j between 1 and k, and then randomly selects j tasks to form a pack.

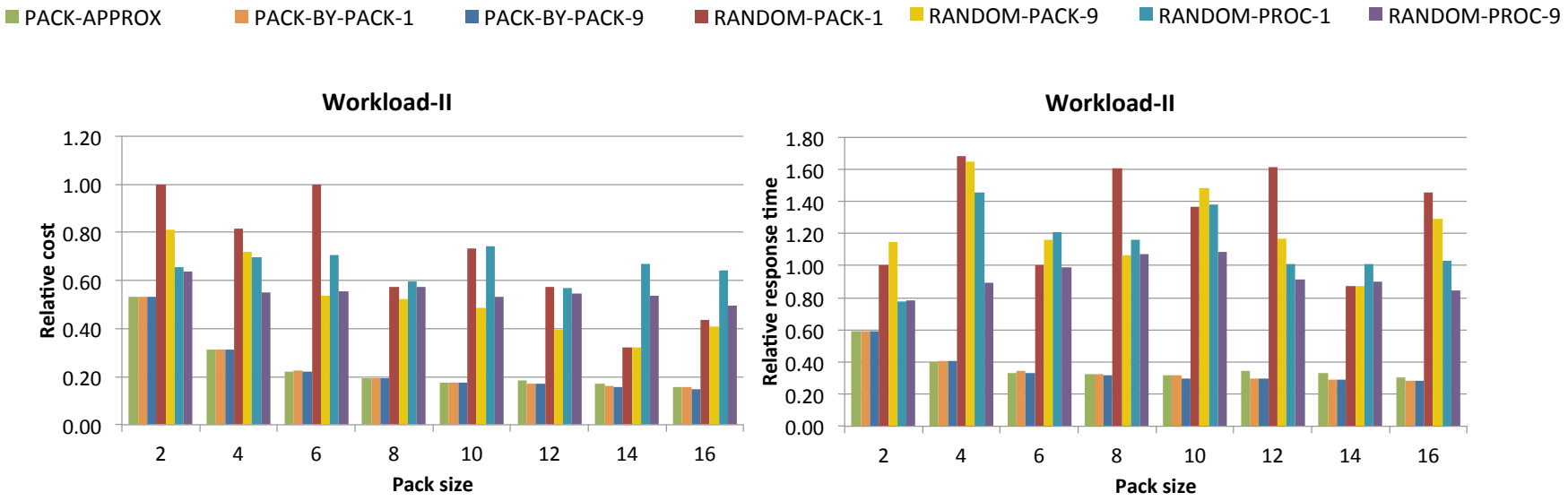
Random-Proc: assigns the number of processors to each task randomly, then calls MAKE-PACK to generate the packs.

pack-by-pack (ε): creates packs that are "well-balanced": the difference between smallest and longest execution times of a pack is small (ratio of $1+\varepsilon$).

pack-Approx: An extension in the case where there are at most k tasks in a pack.

We compare to the heuristics that assigns the maximum number of processors to each task (= scheduler used in practice).

Results

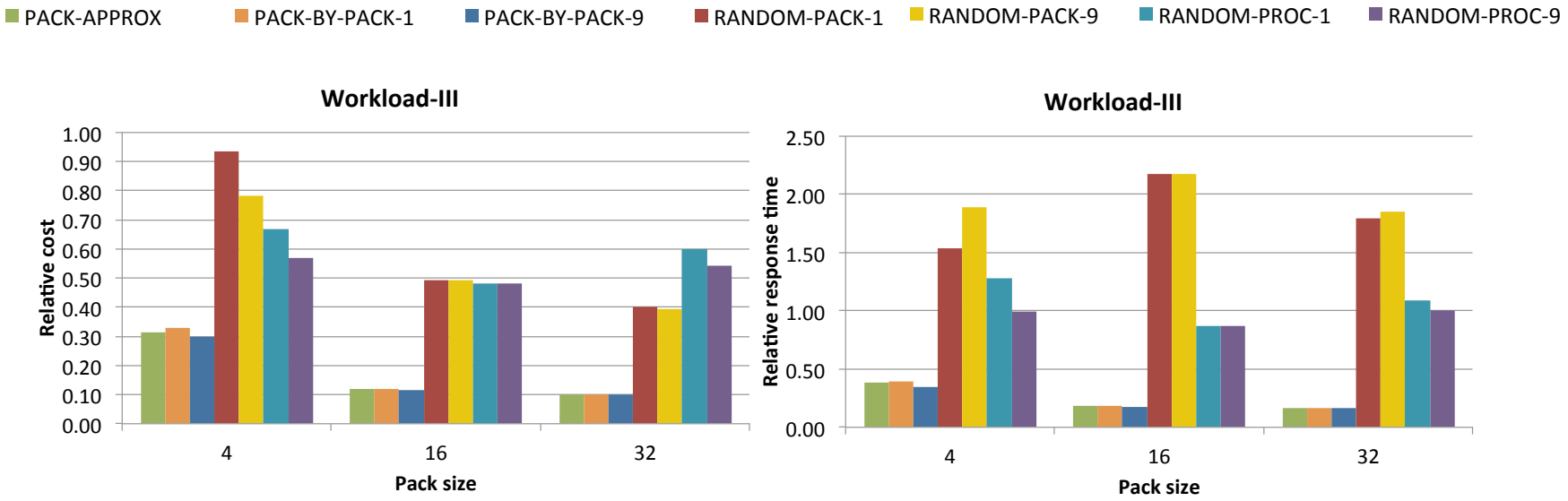


(a) Cost

(b) Response Time

Figure : 65 tasks executing on 16 processors (128 cores).

Results



(a) Cost (b) Response Time

Figure : 260 tasks executing on 32 processors (256 cores).

Conclusion and future work

- *Theoretically:* Hardness + Approximation results
- *Experimentally:* Great improvements compared to existing co-schedulers

Conclusion and future work

- *Theoretically:* Hardness + Approximation results
- *Experimentally:* Great improvements compared to existing co-schedulers

- The main direction is: can we improve the approximation factor? Right now, it does not take pack scheduling into account. We need to bridge the difference "packs/no-packs".
- Approximation on relative response time?
- We need larger simulations.