Analysis of Dynamic Scheduling Strategies for Matrix Multiplication on Heterogeneous Platforms

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Outline

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Scheduling in Large Scale Systems

- Scheduling and ressource allocation problems are hard due to
  - Failures
  - Non-determinism of execution/transfer time
  - Heterogeneity

  All this makes static strategies useless

- Deciding in advance:
  - Where to place tasks
  - When to execute them

  can make the system very slow

Solutions

- On the theoretical side: online scheduling, robust scheduling
  - Given probability distribution of execution/transfer time
  - Find the allocation/schedule that minimizes expected makespan
  - Worst case is also interesting and not trivial
  - Bad point: extremely difficult

- On the practical side:
  - Mostly dynamic, demand driven strategies (e.g. Hadoop, PaRSEC, StarSs, KAAPI, StarPU)
  - Based on rough estimation of execution time
  - Still, open questions:
    - Does it work so well (simulation part) ?
    - Why does it work (theoretical part) ?

On the application side

- More and more applications expressed as independent tasks:
  - MapReduce, Divisible Load
  - Independent tasks are what dynamic schedulers see

- Lessons:
  - Placing tasks close to the actual data location is crucial
  - Placing tasks on well adapted resources is crucial
  - Deciding task order is not crucial, provided that there is no idle time.

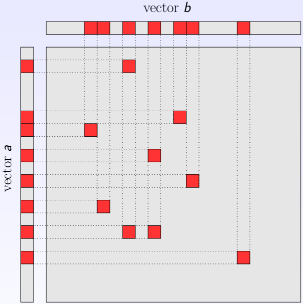
What is our goal?

- Propose analytical models for dynamic strategies
- Analyze dynamic strategies to understand:
  - What makes dynamic strategies efficient?
  - What can be done to improve them?
- For basic applications first
  - Independent tasks, MapReduce
  - Linear algebra without dependencies (except data dep.)
- Today: outer product (and matrix multiplication)

Outline

Outer Product

- Basic $O(N^2)$ operation
  - 2 vectors of size N: a and b
  - Output $M = ab^T$, i.e. all $M_{i,j} = a_i \cdot b_j$ values, $1 \leq i, j \leq N$
- If P_k is responsible for computing the red $M_{i,j}$ values
  it needs to store the red values of a and b



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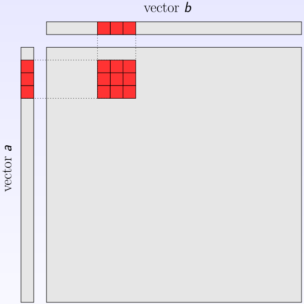


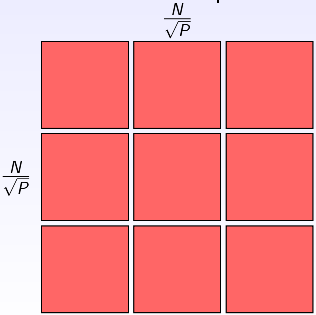
less communications,
same processing

Outer Product : Communication Needs

- Load-balancing ensured by demand-driven strategy
- Overlap communication/computation
- Need to replicate data
- Focus on minimizing communication amount
- Consider first homogeneous processors

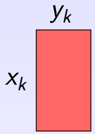
Let us denote by V the total volume of data sent to all processors

- Data sent to processor P_k: $\frac{V}{P}$
- Workload at processor P_k: $\left(\frac{V}{2P}\right)^2$
- Total work is N^2
- So that $P\left(\frac{V}{2P}\right)^2 = N^2$ and $V = 2N\sqrt{P}$
- $\sqrt{P}$ is the smallest achievable replication ratio

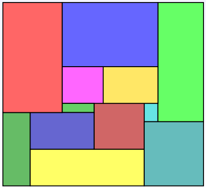


Outer Product : Heterogeneous Processors

- The $\sqrt{P}$ lower bound does not hold in the heterogeneous case
- Let s_k denote the speed of P_k
  - Load balancing, work for P_k:
    $w_k = x_k \times y_k = \frac{s_k}{\sum_i s_i} N^2$
  - Amount of data: $V_k = x_k + y_k$



- Problem: partition the square $[1; N] \times [1; N]$ so that
  - $x_k \times y_k = \frac{s_k}{\sum_i s_i} N^2$
  - $\sum_k x_k + y_k$ is minimized
- Lower bound: $2N \sum_k \sqrt{\frac{s_k}{\sum_i s_i}}$
- Well known combinatorial problem
  - 7/4-approximation algorithm



Outer Product: Need for Dynamic Strategies

- Previous approximation algorithm:
  - Very static
  - Not well suited to large-scale dynamic faulty systems
  - Processor speeds are hard to predict
- More dynamic strategies are needed
  - Demand-driven strategies
  - Processors request blocks when needed (or slightly before)
  - We assume there is a central scheduler/dispatcher
  - Question: which task to send to which processor?

Randomized Dynamic Strategies

Basic strategies:

- When requested for a new task, send a random one (RandomOuter)
- Send tasks by rows (SortedOuter)

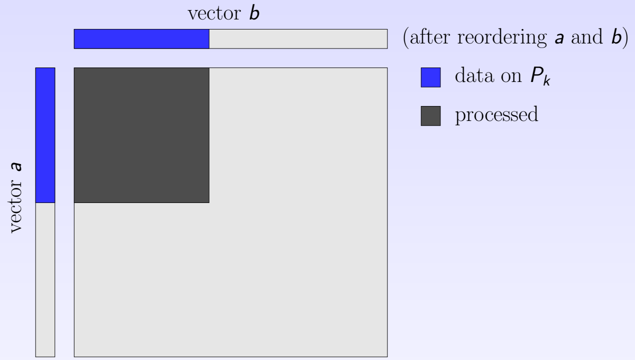
Expect to lead to large amount of communications, because of data replication

Simple data-aware strategy DynamicOuter:

- When a processor P_k request a task, send new a_i and b_j to P_k
- Allocate all unprocessed tasks $a_i \times b_{j'}$ (for $b_{j'}$ already on P_k)
- Allocate all unprocessed tasks $a_{i'} \times b_j$ (for $a_{i'}$ already on P_k)

(small scheduling overhead: maintain sets of a and b on each processors)

Randomized Dynamic Strategies



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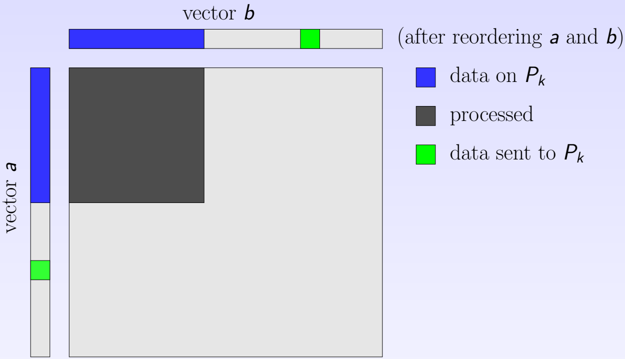

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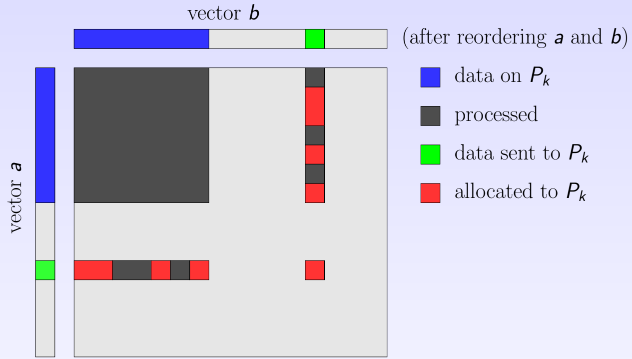

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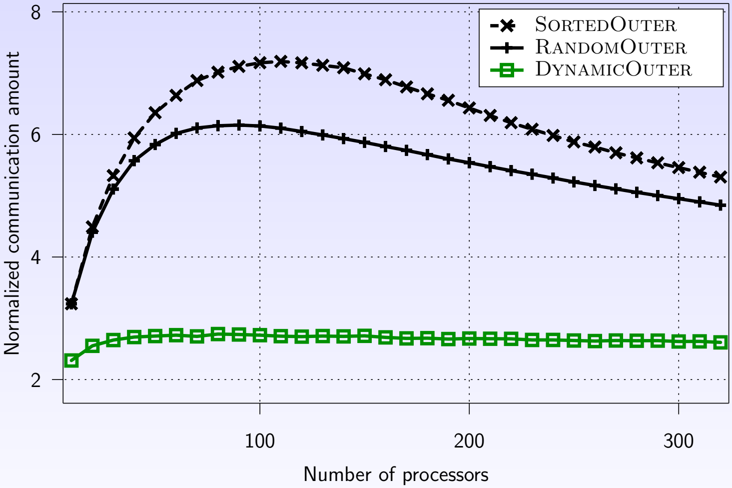
Basic Dynamic Strategies – Comparison



- $N/l = 100$ blocks
- Normalized by the lower bound

DynamicOuter: Shortcomings

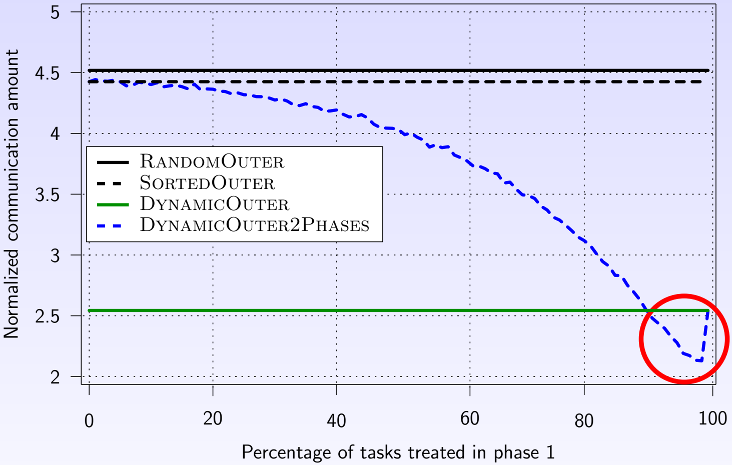
Limitation:

- When few tasks remains to be processed
- Needs to send many data blocks before reaching these tasks
- Induce useless communication

New strategy: DynamicOuter2Phases

- Start with DynamicOuter
- Switch to the random strategy at the end:
  - Allocate any unprocessed task
  - Send 2 corresponding data blocks
- When the number of unprocessed tasks is below a given threshold

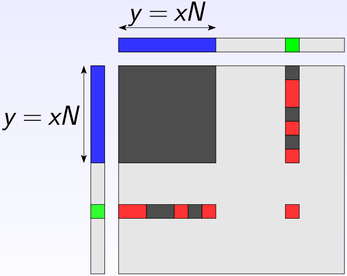
DynamicOuter2Phases: Threshold



- $P = 20$ processors, $N/l = 100$ blocks
- Allows to reduce communications even more
- Optimal threshold: a few percent
- How to determine this threshold ?

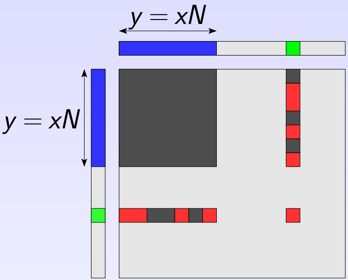
DynamicOuter2Phases: Analysis

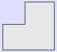
- Assume that the size N of both vectors is large
- Consider a continuous dynamic system with close behavior
- Describe the continuous system using Ordinary Differential Equations
- Approximate the randomized discrete process by the continuous system (no proof)



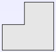
- Ratio $x = y/N$ of elements of a (and b) on P_k at $t_k(x)$
- Basic step: when this ratio goes from x to $x + \delta x = y/N + \ell/N$
- In $\blacksquare$: all tasks processed (by P_k or other processors)

Simple Data-Aware Strategy: Analysis



- In  : $g_k(x)$ is the fraction of unprocessed tasks (assumed uniformly distributed)
- Time for P_k to compute the red tasks:

$$\frac{2x\ \delta x\ g_k(x) N^2}{s_k} = t_k(x+\delta x) - t_k(x)$$

- Number of tasks from  computed by other processors during this step: $(t_k(x+\delta x) - t_k(x)) \sum_{i \neq k} s_i$
- Evolution of $g_k(x)$:

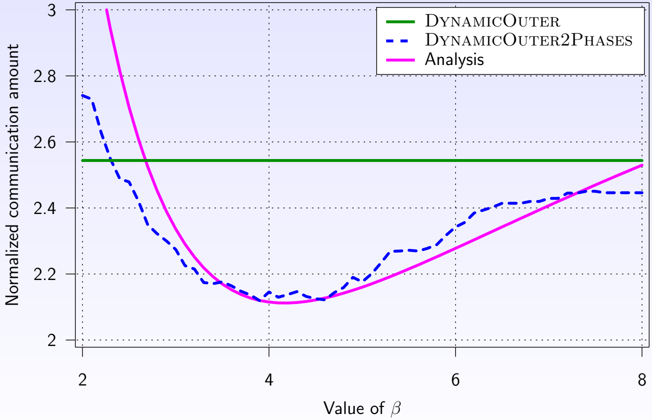
$$g_k(x+\delta x) - g_k(x) = g_k(x) \delta x \frac{-2x\alpha_k}{1-x^2} \quad \text{where } \alpha_k = \frac{\sum_{i \neq k} s_i}{s_k}$$

$$\Rightarrow \quad g_k(x) = (1-x^2)^{\alpha_k}$$

Simple Data-Aware Strategy: Analysis

- From $g_k(x)$, it is possible to compute $t_k(x)$, the total communication amount and when to switch to the random strategy
- Comparison discrete simulation vs. continuous analysis:

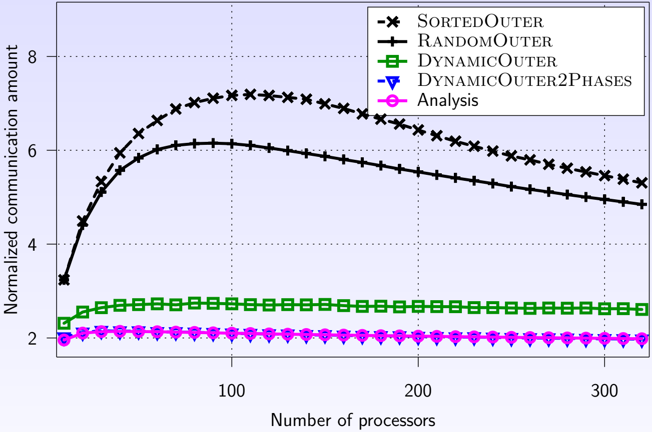
$\Rightarrow$



- β: parameter that defines the threshold

Simulations

Comparison with previous heuristics:



- β has a very small deviation with the speed distribution
- Runtime estimation of β: use homogeneous speeds

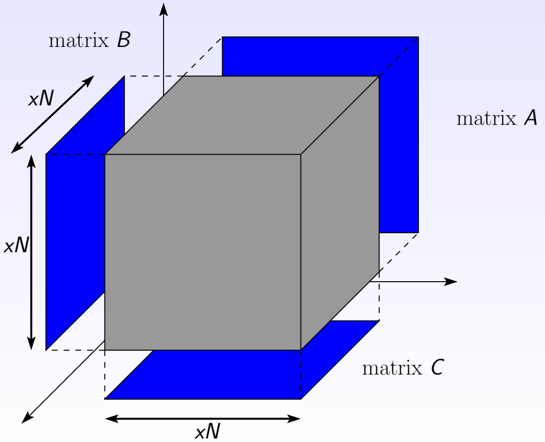
Outline

Matrix-Product $C = AB$

- Very similar problem: from 2D to 3D
- Basic computation task: $C_{i,j} \leftarrow C_{i,j} + A_{i,k} B_{k,j}$
- Send elements of A and B, gather elements from C
- A, B and C need to be replicated
- Minimize communication amount

Matrix-Product: Simple Data-Aware Strategy

- Adapt the previous heuristic:
  - P_k knows $xN \times xN$ values of A, B and C
  - When P_k requests some work
  - Send $2x-1$ data blocks of A, B, C
  - Allocate all tasks available with these new data



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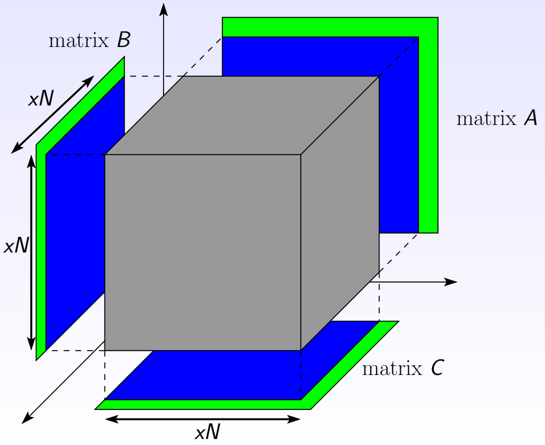

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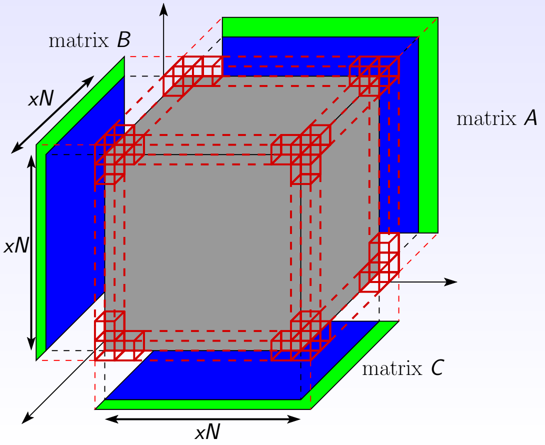
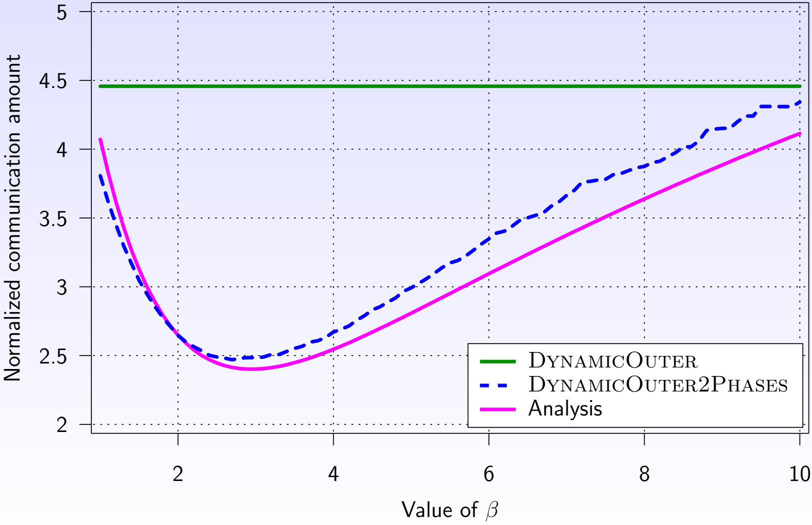
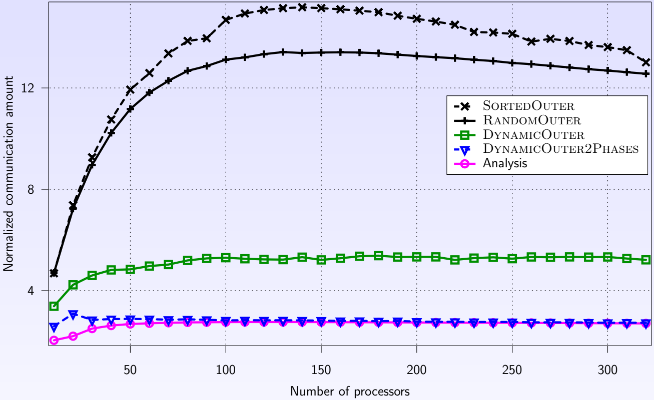


Matrix-Product: Threshold

Similar analysis allows to compute when to switch to the random strategy



Matrix-Product: Simulations



- $N/l = 100$ blocks ($N^3/l^3 = 1,000,000$ tasks)

Outline

Conclusion and Perspectives

What we have done:

- Design and analyze a simple randomized dynamic strategy
- Analysis allows to tune the strategy (threshold)
- For outer-product and matrix-product (mainly independent tasks)

What we should/will do:

- Convergence proof (Mean Field Approximation?)
- Analysis in a dynamic environment
- Mix static and dynamic strategy
- Move to more complex kernels
  - Cholesky decomposition

Thank You !

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