Multidimensional Assignment Problems for Semiconductor Plants

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About merging vectors

Our problem - a prologue

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- Let $u = (12\ 91\ 7)$, and $v = (47\ 32\ 12)$.
- How do we merge u and v?
- Well, we say that
  $u \vee v = (\max(u_1, v_1), \max(u_2, v_2), \max(u_3, v_3)) = (47\ 91\ 12)$

Oh, and the *cost* of a vector is represented by a function $c(u) : \mathbb{Z}_+^p \to \mathbb{R}_+$.

Our Problem

Instance:

- m sets: $V_1, V_2, \ldots, V_m$
- Each V_i consists of n vectors each of size p, $1 \leq i \leq m$
- Each entry of a vector is a non-negative integer

Objective:

- partition the given m sets into n m-tuples, such that each m-tuple contains one vector from each set V_i
- *minimize* the total cost of this partition

We will abbreviate the name of this problem as MVA.

An instance of our problem MVA

Let $m = 3$, and let the three sets be denoted by V_1, V_2, and V_3. The length of each vector, p, equals 3, and $n = 4$, and let us specify c as the sum of the entries of a vector, ie, $c(u) = \sum_{i=1}^{p} u_i$.

V_1	V_2	V_3
(12 91 7)	(47 31 12)	(83 3 37)
(54 29 64)	(5 44 73)	(37 2 80)
(92 32 26)	(40 15 71)	(38 13 68)
(2 97 43)	(32 32 32)	(12 91 7)

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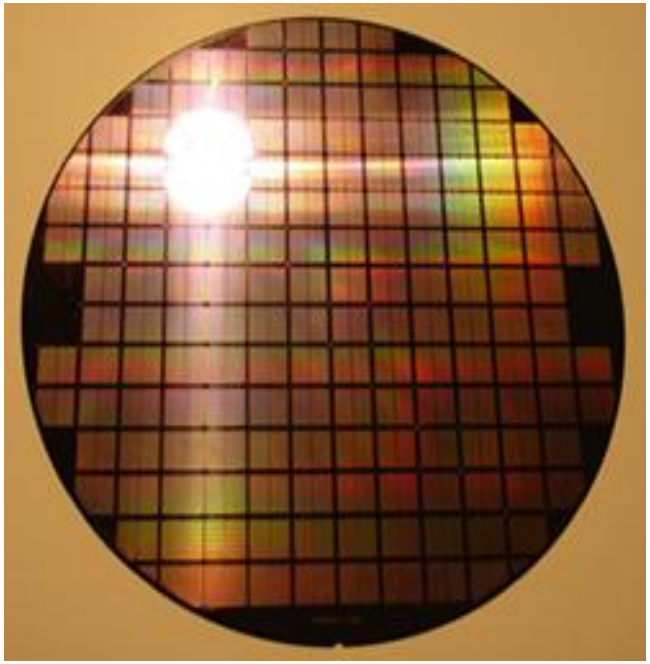
A particular m-tuple could consist of the second vector of V_1 ((54 29 64)), the first vector of V_2 ((47 31 12)), and the fourth vector of V_3 ((12 91 7)), coming out at: (54 91 64).

A wafer



Si Wafer

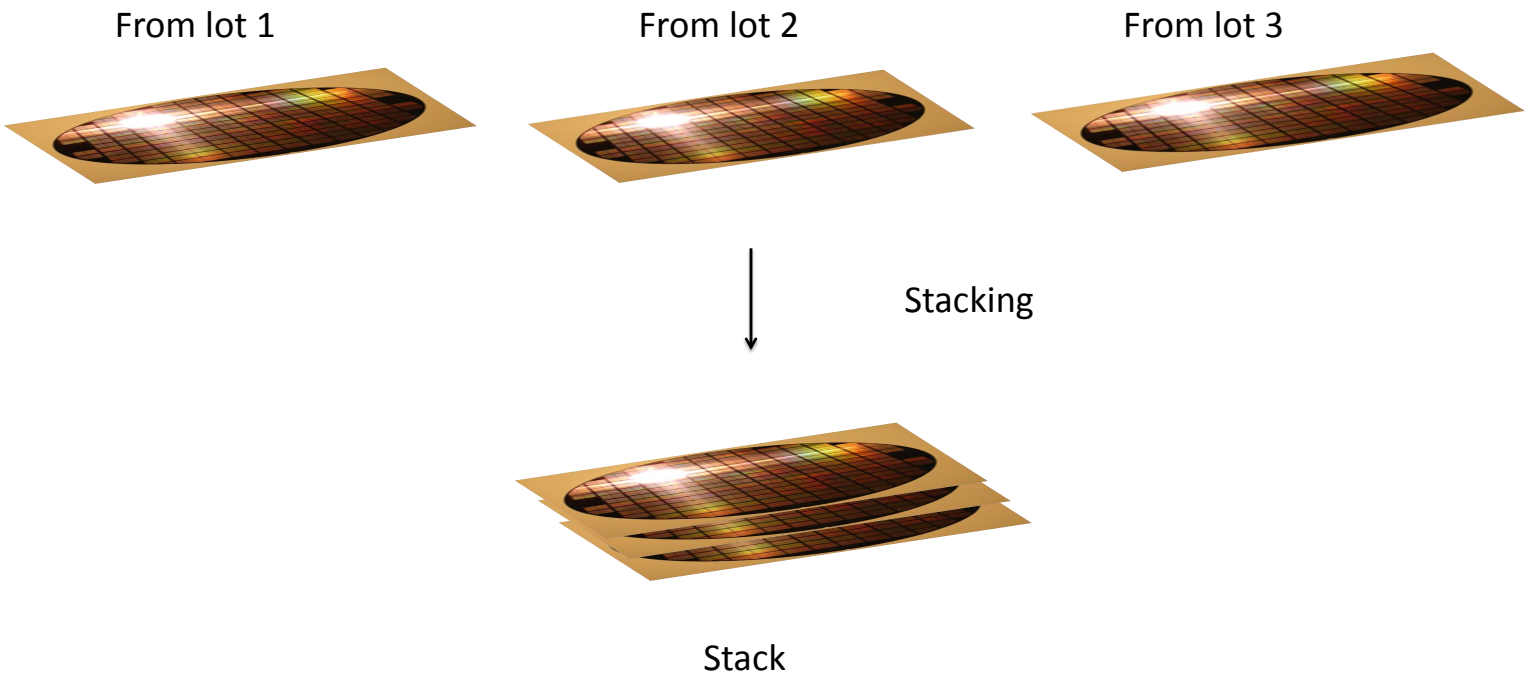
Emerging Technology

Through Silicon Vias(TSV) based Three-Dimensional Stacked Integrated Circuits (3D-SIC)

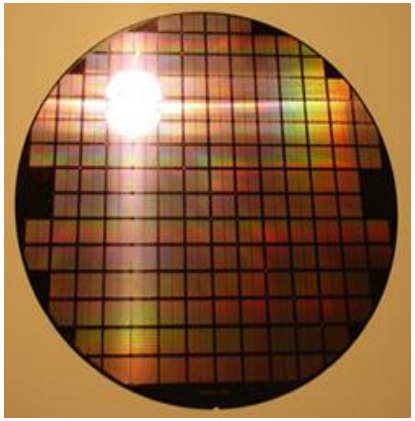
Benefits

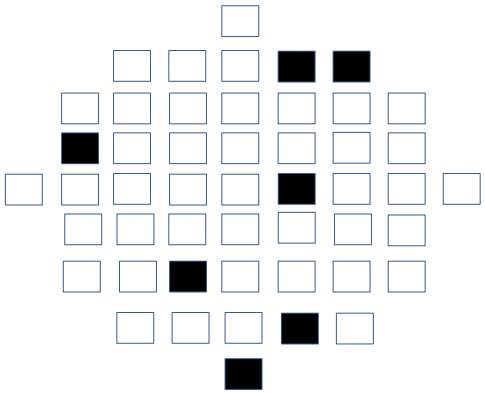
- smaller footprint
- higher interconnect density
- higher performance
- lower power consumption compared to planar IC's

Stacking wafers



Yield optimization: bad dies and good dies

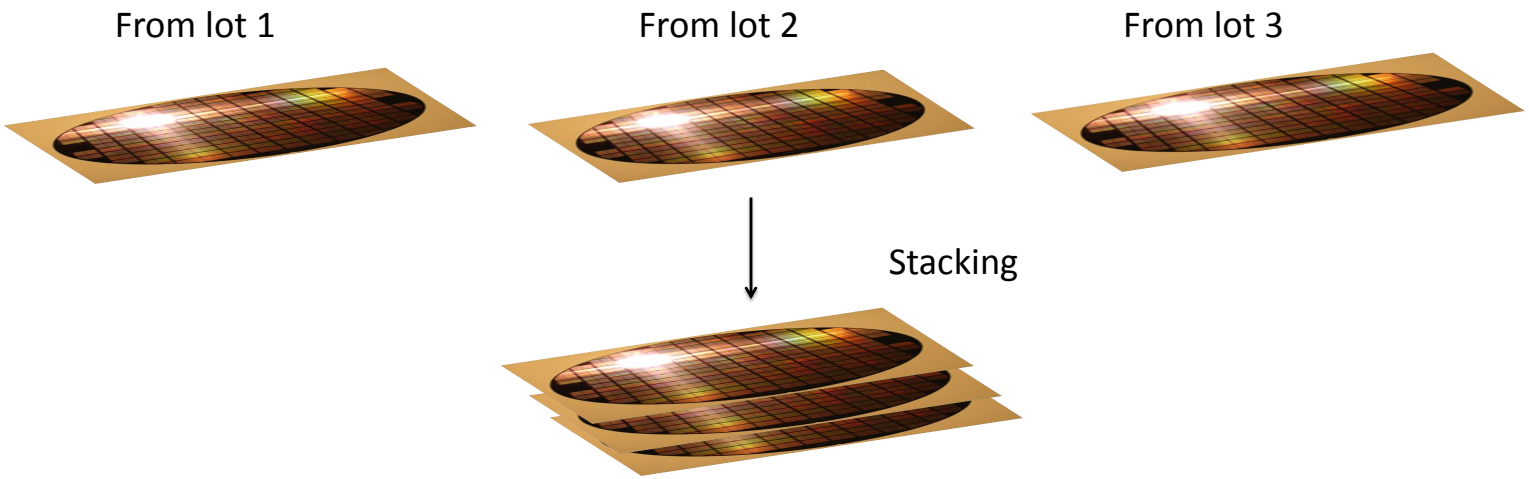




(0,..,0,1,1,0,...0,1,0,...,0,1,0,...,0,1,0,...,0,1,0,1)

Defect map

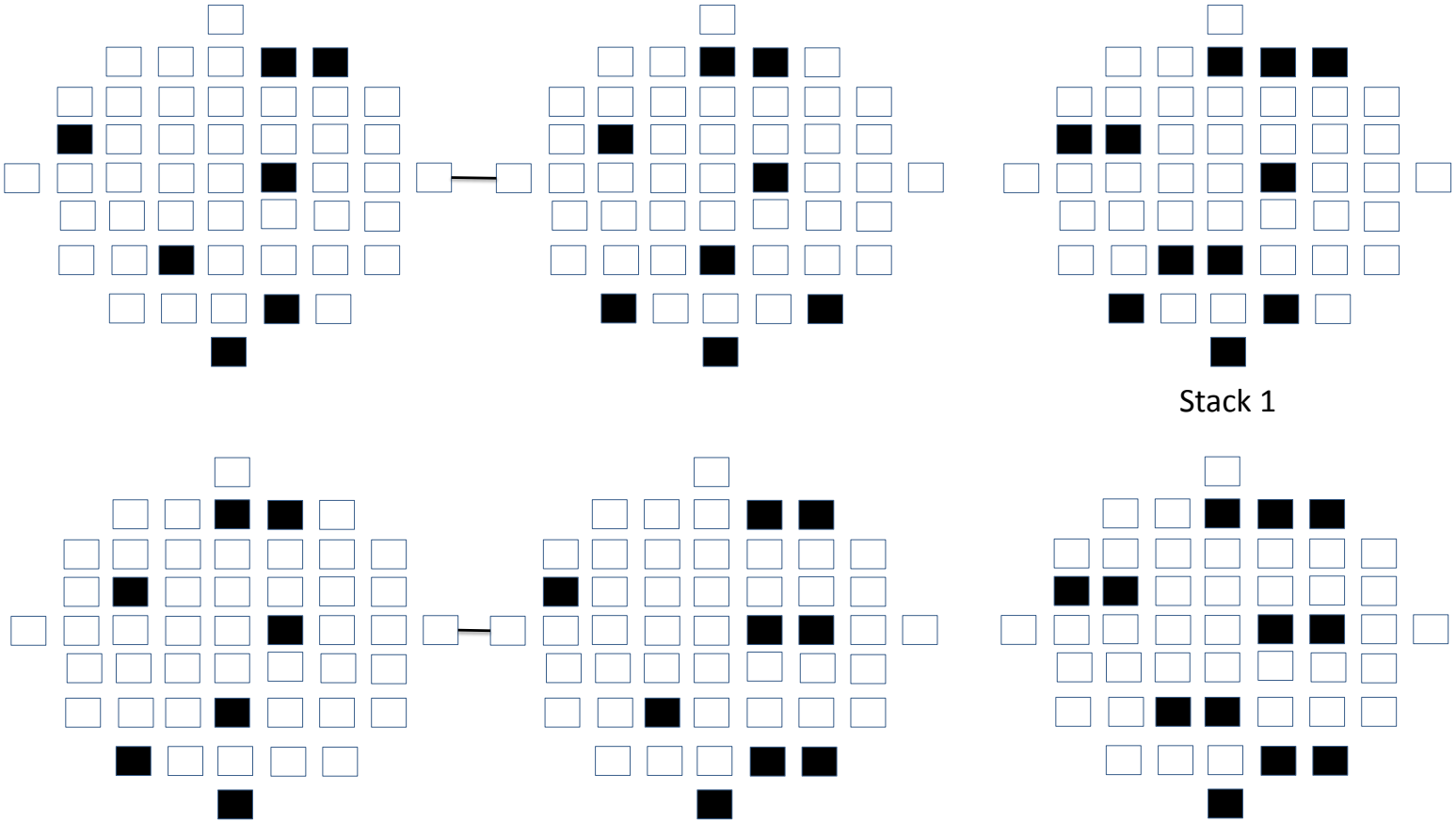
Yield optimization: superimposing dies



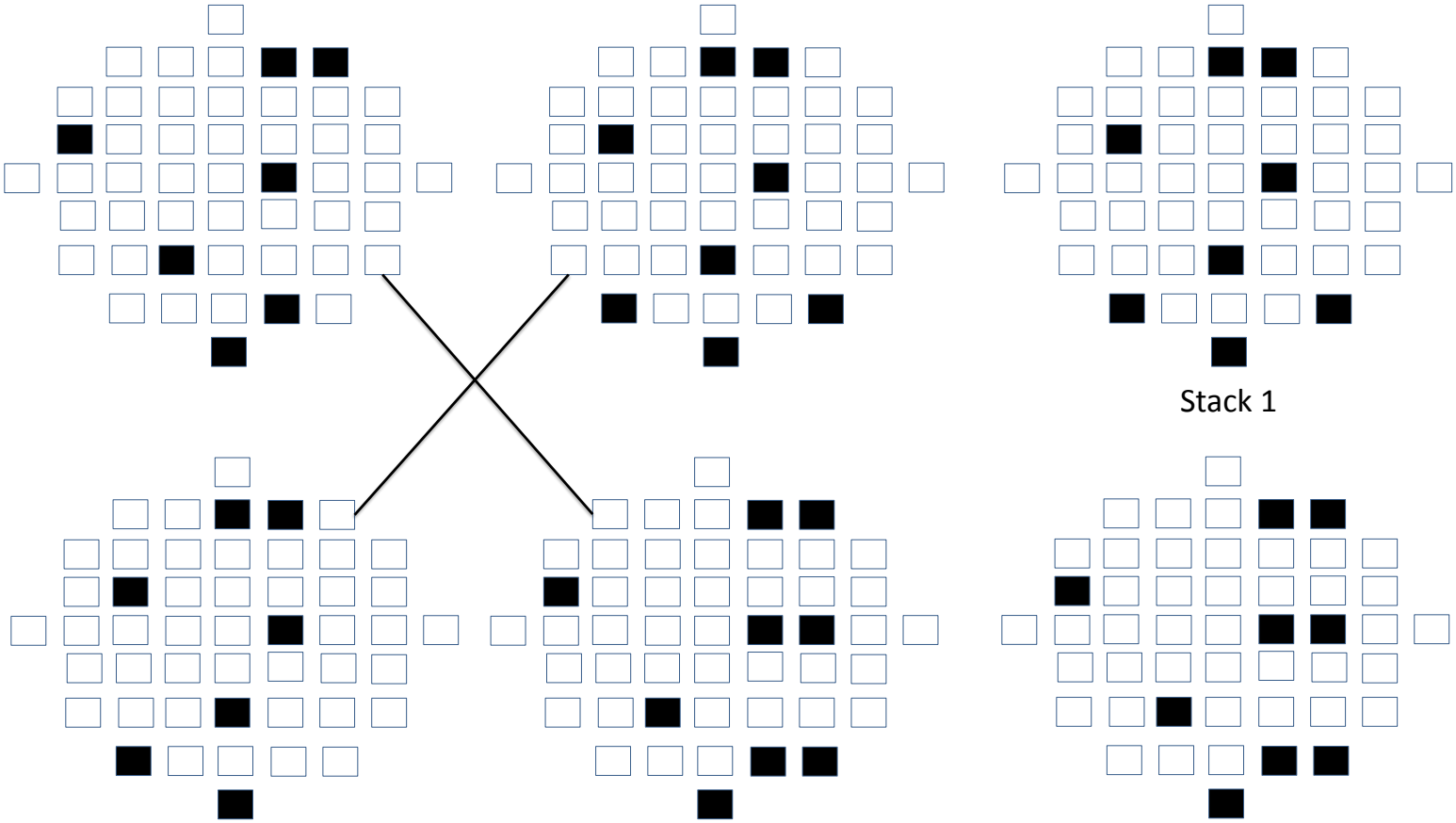
Defect map of resulting stack:
(0,..,0,1,1,0,...0,1,0,...,0,1,0,...,0,1,0,...,0,1,0,1)

Yield = no. of zeros in defect map vector

Yield optimization: an example



Yield optimization: an example



Total number of bad dies in stack 1 + stack 2 = 17

Previous work

- S. Reda, L. Smith, and G. Smith. Maximizing the functional yield of wafer-to-wafer integration. IEEE Transactions on VLSI Systems, 17:13571362, 2009.
- M. Taouil and S. Hamdioui. Layer redundancy based yield improvement for 3D wafer-to-wafer stacked memories. IEEE European Test Symposium, pages 4550, 2011.
- M. Taouil, S. Hamdioui, J. Verbree, and E. Marinissen. On maximizing the compound yield for 3D wafer-to-wafer stacked ICs. In IEEE, editor, IEEE International Test Conference, pages 183192, 2010.
- J. Verbree, E. Marinissen, P. Roussel, and D. Velenis. On the cost-effectiveness of matching repositories of pre-tested wafers for wafer-to-wafer 3D chip stacking. IEEE European Test Symposium, pages 3641, 2010.
- Eshan Singh. Wafer ordering heuristic for iterative wafer matching in w2w 3d-sics with diverse die yields. In 3D-Test First IEEE International Workshop on Testing Three-Dimensional Stacked Integrated Circuits, 2010. poster.

Yield optimization is a special case of MVA

Observe that in the yield optimization application, all vectors are $\{0, 1\}$-vectors, and that the cost-function c is additive, ie, $c(u) = \sum_{i=1}^{p} u_i$. Instances from practice may have $m = 10$, $n = 75$, and $p = 1000$.

We refer to this special case of MVA as the *Wafer-to-Wafer Integration* problem (WWI).

Cost Functions

Monotonicity

If $u, v \in Z_+^p$ and $u \leq v$, then $0 \leq c(u) \leq c(v)$.

Subadditivity

If $u, v \in Z_+^p$, then $c(u \vee v) \leq c(u) + c(v)$.

Submodularity

If $u, v \in Z_+^p$, then $c(u \vee v) + c(u \wedge v) \leq c(u) + c(v)$.

Modularity

If $u, v \in Z_+^p$, then $c(u \vee v) + c(u \wedge v) = c(u) + c(v)$.

Heuristics

- Sequential Heuristics
  - Sequential Heuristic (H^{seq}): Solve a bipartite assignment problem between H_{i-1} and V_i. Let H_i be the resulting assignment for $V_1 \times \ldots \times V_i$; $i = 2, \ldots, m$. Return H_m.
  - Heavy Heuristic (H^{heavy}): Rearrange the sets such that $c(V_1)$ is the heaviest. Apply H^{seq}.
- Hub Heuristics
  - Single-hub Heuristic (H^{shub}): Choose a hub $h \in \{1, \ldots, m\}$. Solve an assignment problem between V_h and V_i (call the resulting solutions M_{hi}). Construct a feasible solution by combining the solutions M_{hi}.
  - Multi-hub Heuristic (H^{mhub}): Apply H^{shub} for each possible choice of hub and output the best solution among all.

Example

V_1	V_2	V_3
00	00	10
01	10	01

Example: the optimum

V_1	V_2	V_3
00	00	10
01	10	01

Results

Results Overview

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- When c is additive, the Heaviest-first has a better performance: $\rho^{heavy}(m) \leq \frac{1}{2}(m+1) - \frac{1}{4}\ln(m-1)$.

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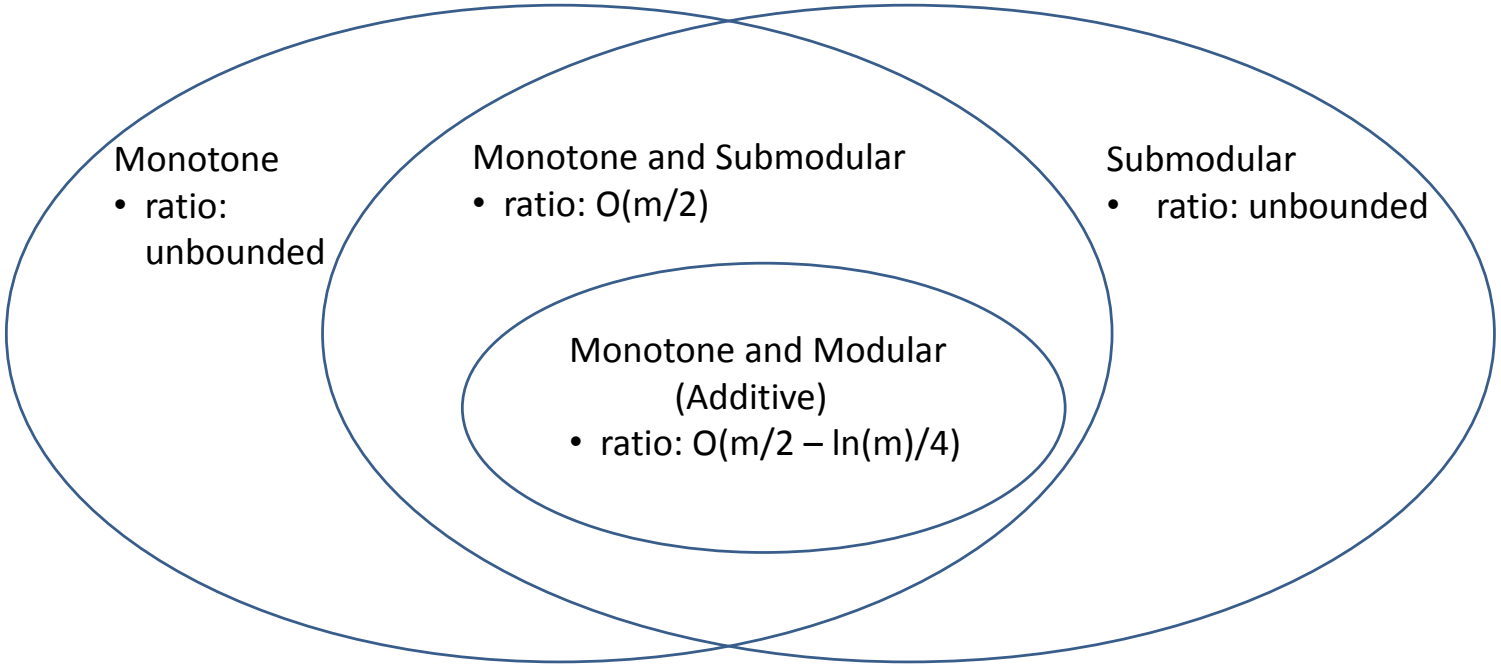
- When c is additive, the Heaviest-first has a better performance: $\rho^{heavy}(m) \leq \frac{1}{2}(m+1) - \frac{1}{4}\ln(m-1)$.
- WWI-3 is APX-hard.
- WWI with fixed p is solvable in polynomial time.

Overview of results



Monotone and Submodular Case

Analysis of H^{seq}

Notation:

- $c(H_r)$ = value of partial solution restricted to $V_1 \times \ldots V_r$,
- $c(A_{m-2,m})$ = value of the partial solution corresponding to an optimal assignment between H_{m-2} and V_m,
- $c(V_i)$ = total weight of the set V_i, $i = 1, \ldots, m$.

Analysis of Heuristic H^{seq}

- Recall: $A_{m-2,m}$ = solution of optimal assignment between V_m and H_{m-2}

Case 1: $c(V_{m-1}) \leq \frac{1}{2} c_m^{OPT}$

$$c(H_m) \leq c(A_{m-2,m}) + c(V_{m-1})$$

$$c(A_{m-2,m}) \leq \frac{1}{2}(m-1)\, c^{OPT}(W) \leq \frac{1}{2}(m-1)\, c_m^{OPT}$$

where $W = V_1 \times \ldots \times V_{m-2} \times V_m$

$$c(H_m) \leq (\frac{m-1}{2} + \frac{1}{2})\, c_m^{OPT} = \frac{m}{2}\, c_m^{OPT}.$$

Analysis of Heuristic H^{seq}

- $M_{m-1,m}$ = solution of optimal assignment between V_{m-1} and V_m

Case 2: $c(V_{m-1}) \geq \frac{1}{2} c_m^{OPT}$

$$
\begin{aligned}
c(H_m) \quad & \leq c(H_{m-1}) + c(M_{m-1,m}) - c(V_{m-1}) \\
& \leq \frac{m-1}{2} \cdot c_{m-1}^{OPT} + c_m^{OPT} - \frac{1}{2} \cdot c_m^{OPT} \\
& \leq \left(\frac{m-1}{2} + \frac{1}{2}\right) c_m^{OPT} \\
& \leq \frac{m}{2} c_m^{OPT}
\end{aligned}
$$

Analysis of Heuristic H^{seq}: Example

Theorem

When the cost-function c is monotone and submodular, the sequential heuristic has a performance ratio of $\rho^{seq}(m) = \frac{1}{2}m$. This bound is tight even when the input of MVA-m is restricted to binary vectors.

Tight example

- $c(u) = f(\sum_{i=1}^{p} u_i)$, where $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x$ when $x \leq 2$, and $f(x) = 2$ when $x \geq 2$.
- f is monotone nondecreasing and concave, and c is monotone and submodular.
- $p = n = m - 1$, $V_i = \{e_i, \mathbf{0}, \ldots, \mathbf{0}\}$ for $i = 1, \ldots, m$, where e_i is the ith unit vector.
- $c(H_m) = m$ and $c_m^{OPT} = 2$.

Hardness

Theorem

WWI-3 is APX-hard even when all vectors in $V_1 \cup V_2 \cup V_3$ are $\{0, 1\}$ vectors with exactly two nonzero entries per vector.

Sketch

- L-reduction from 3-bounded MAX-3DM to WWI-3.

Binary inputs and fixed p case

Theorem

Binary MVA can be solved in polynomial time for each fixed p.

Binary MVA - MIP

A mixed integer formulation of MVA with variables:
for each $t = 1, \ldots, 2^p$,

$$x_t = \text{number of } m\text{-tuples of type } t \text{ in the assignment, .}$$

for each $i = 1, \ldots, m; j = 1, \ldots, n; t = 1, \ldots, 2^p$,

$$z_{jt}^i = 1 \text{ if } v_{ij} \text{ is assigned to an } m\text{-tuple of type } t.$$

Binary inputs and fixed p case

Binary MVA - MIP

$$\min \sum_{t=1}^{2^p} c(b_t)x_t \tag{1}$$

$$\sum_{j:\ b_t \geq v_{ij}} z_{jt}^i = x_t \qquad \text{for each } t, i \tag{2}$$

$$\sum_{t:\ b_t \geq v_{ij}} z_{jt}^i = 1 \qquad \text{for each } j, i \tag{3}$$

$$x_t \quad \text{integer} \qquad \text{for each } t \tag{4}$$

$$z_{jt}^i \geq 0 \qquad \text{for each } j, t, i. \tag{5}$$

Claim: Binary MVA - MIP can be solved in polynomial time for every fixed p.

Future work and extensions

Questions

1. What is the exact approximation ratio of the multi-hub heuristic in case of additive costs? We know that it lies between $m/4$ and $m/2$.
2. What is the exact approximation ratio of the heaviest-first sequential heuristic in case of additive costs? We know that it lies between $\Omega(\sqrt{m})$ and $O(m - \ln m)$.
3. Does there exist a polynomial-time algorithm with constant (i.e., independent of m) approximation ratio for MVA-m?
4. Can we design practical exact algorithm based on Binary MVA - MIP for reasonable n,p and m?

Europe

MAPSP2015 takes place in La Roche, 2015, June 8 - June 12

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THANKS!