New Challenges in Scheduling Theory
Aussois

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Optimizing Supply Process in Charitable Organizations by Genetic Algorithm

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Scope of the Talk

- Problem definition
- MP Formulation
- Heuristic Algorithm
- Genetic Algorithm
- Computational Experiments
- Conclusions

Problem Definition

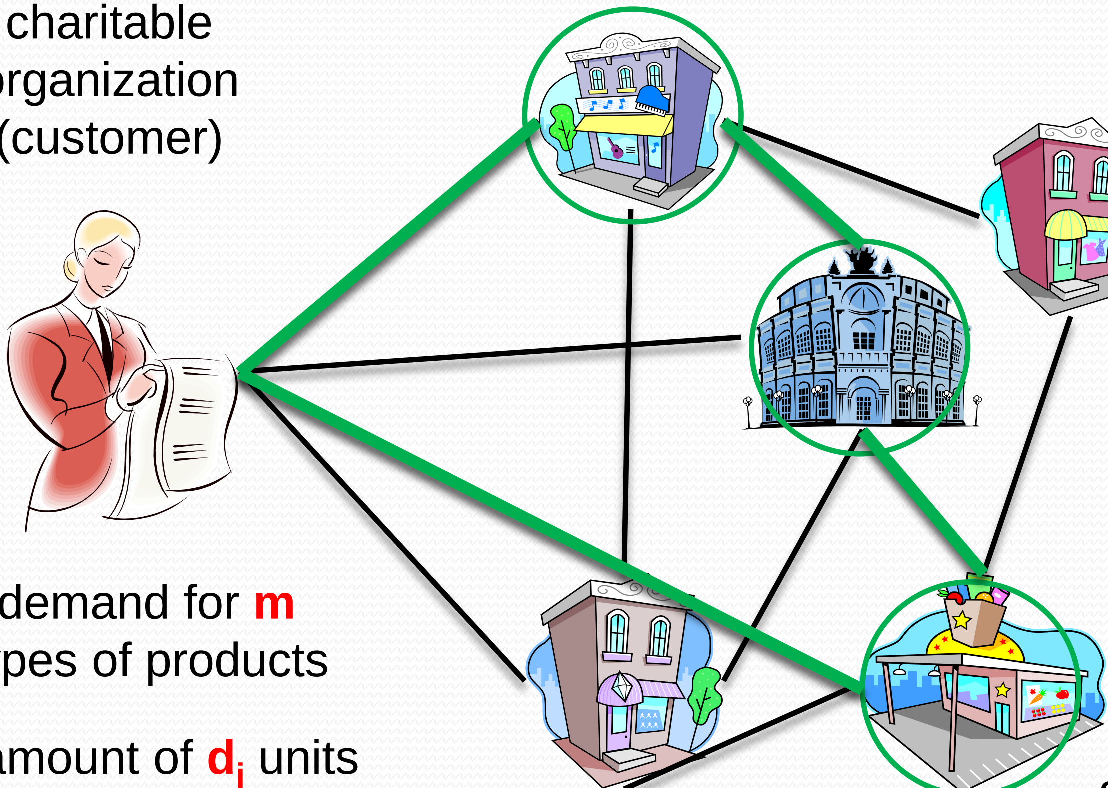
- web service for charitable organizations
- gathering charitable organizations and donors
- allowing submitting requests and registering offers for various products

- managing database for registered users
- supporting supply process with optimization algorithms

Problem Definition

products are offered by **n** depots (donors, warehouses, shops)

charitable organization (customer)



in amount of a_{ij} units

at price c_{ij}

unit transportation cost **T**

distance t_{ir}

demand for **m** types of products

in amount of d_j units

position of a depot in the route y_{ik}

amount of ordered units x_{ij} of particular products

MP Formulation

$$\min \sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}c_{ij} + T\left(\sum_{j=1}^{n} y_{i1}t_{0i} + \sum_{i=1}^{n}\sum_{r=1}^{n}\sum_{k=1}^{n-1} \max\{0, y_{ik} + y_{r,k+1} - 1\}t_{ir} + \sum_{i=1}^{n}\sum_{r=1}^{n}\left(y_{ik}\left(1 - \sum_{r=1}^{n} y_{r,k+1}\right)\right)\right)$$

under constraints

order delivery subproblem

$$\sum_{i=1}^{n} x_{ij} = d_j \qquad j = 1 \ldots m$$

$$x_{ij} \le a_{ij} \qquad i = 1 \ldots n, j = 1 \ldots m$$

$$x_{ij} \ge 0 \text{ and integer} \qquad i = 1 \ldots n, j = 1 \ldots m$$

order completion subproblem

$$\sum_{k=1}^{n} y_{ik} = \min\{1, \sum_{j=1}^{m} x_{ij}\} \qquad i = 1 \ldots n$$

$$\sum_{k=1}^{n} y_{ik} \le 1 \qquad i = 1 \ldots n$$

$$\sum_{i=1}^{n} y_{ik} \le 1 \qquad k = 1 \ldots n$$

$$\sum_{i=1}^{n} y_{i,k+1} \le \sum_{i=1}^{n} y_{ik} \qquad k = 1 \ldots n-1$$

$$y_{ik} \in \{0,1\} \qquad i = 1 \ldots n, k = 1 \ldots n+1$$

Problem Formulation

- selecting depots offering demanded products at the lowest prices
  - **order completion**
  - „easy” problem – greedy solution is optimal
- determining the shortest route to pick up products from these depots
  - **order delivery**
  - „hard” problem – reduces to the shortest Hamiltonian cycle
- order completion and delivery is strongly NP-hard as a variant of **Travelling Purchaser Problem** (Ramesh 1981)

Heuristic Algorithm

- two-phase heuristic approach
  - selecting depots
  - constructing a tour

Heuristic Algorithm

- selecting depots - choosing depots until demand is satisfied
- ordering depots according to
  - product cost (**greedy heuristic**)
  - weighted priorities (**priority heuristic**), based on:
    - total distance to other locations
    - total cost of demanded products available at a depot
    - total cost of all demanded products

  various priority weights result in
  various list heuristics and
  various sets of selected depots

Heuristic Algorithm

- constructing a tour from selected depots
- Minimum Spanning Tree Heuristic (Hedl & Karp 1970)
  - constructing minimum spanning tree by Kruskal Algorithm
  - traversing the tree according to Depth First Search Strategy
  - converting DFS sequence to the Hamiltonian cycle

- MSTH is 2-approximation algorithm

Bounds

- heuristic solution determines upper bound (UB)
- lower bound

$$LB = \sum_{j=1}^{m} \sum_{i=1}^{\tilde{n}} x_{\pi_i,j} c_{\pi_i,j} + T(\min_{i=1\ldots n} \{t_{0i}\} + \min_{i=1\ldots n} \{t_{i0}\})$$

- reference bounds

$$RB = \sum_{j=1}^{m} \sum_{i=1}^{\tilde{n}} x_{\pi_i,j} c_{\pi_i,j} + T\,(\min_{i=1\ldots n}\{t_{0i}\} + \sum_{k=1}^{\tilde{n}-1} t_{[k]} + \min_{i=1\ldots n} \{t_{i0}\})$$

$t_{[k]}$ - k'th distance between depots

Genetic Algorithm

- solution is a sequence of assignments:
  - number of product units ordered from a depot
  - one product can be taken from more than one depot

  (to determine a complete solution a tour is constructed by MST heuristic)
- initial population
  - heuristic solutions corresponding to various priorities weights
  - random solutions
- new population replaces the previous one

Genetic Algorithm – Operators

- one-point crossover and two-point crossover
  - exchanging parts of assignments product-depot
  - infeasible offspring repair procedure:
    - exceeding the product availability at a certain depot
    - taking products from another depot in offspring
    - taking products from a depot in parental solution

- mutation
  - replacing a given number of assignments (product-depot) with random assignment

Genetic Algorithm - Selection

- selecting mating population according to crossover rate
  - roulette selection

    (according to criterion values)
  - ranking selection

    (according to the position in ranking)
  - tournament selection

    (the best solution from randomly chosen groups)

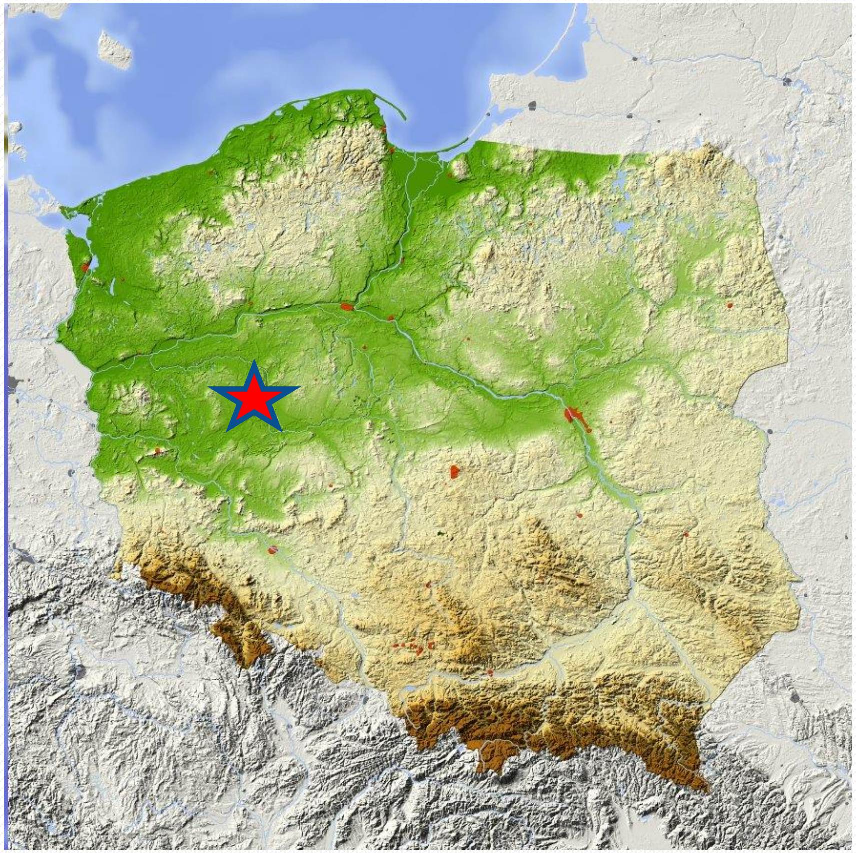
- mutation according to mutation rate

Genetic Algorithm – Termination Condition

- the maximum number of generations
- the maximum number of generations without improvement
- exceeding the satisfying ratio of the criterion value improvement

Computational Experiments

- random instances reflecting real world scenarios
- depots located in 48 Polish cities
- the charitable organization located in Poznań
- distances correspond to road distances (Bing Maps)
- unit transportation cost determined by government regulations



Computational Experiments

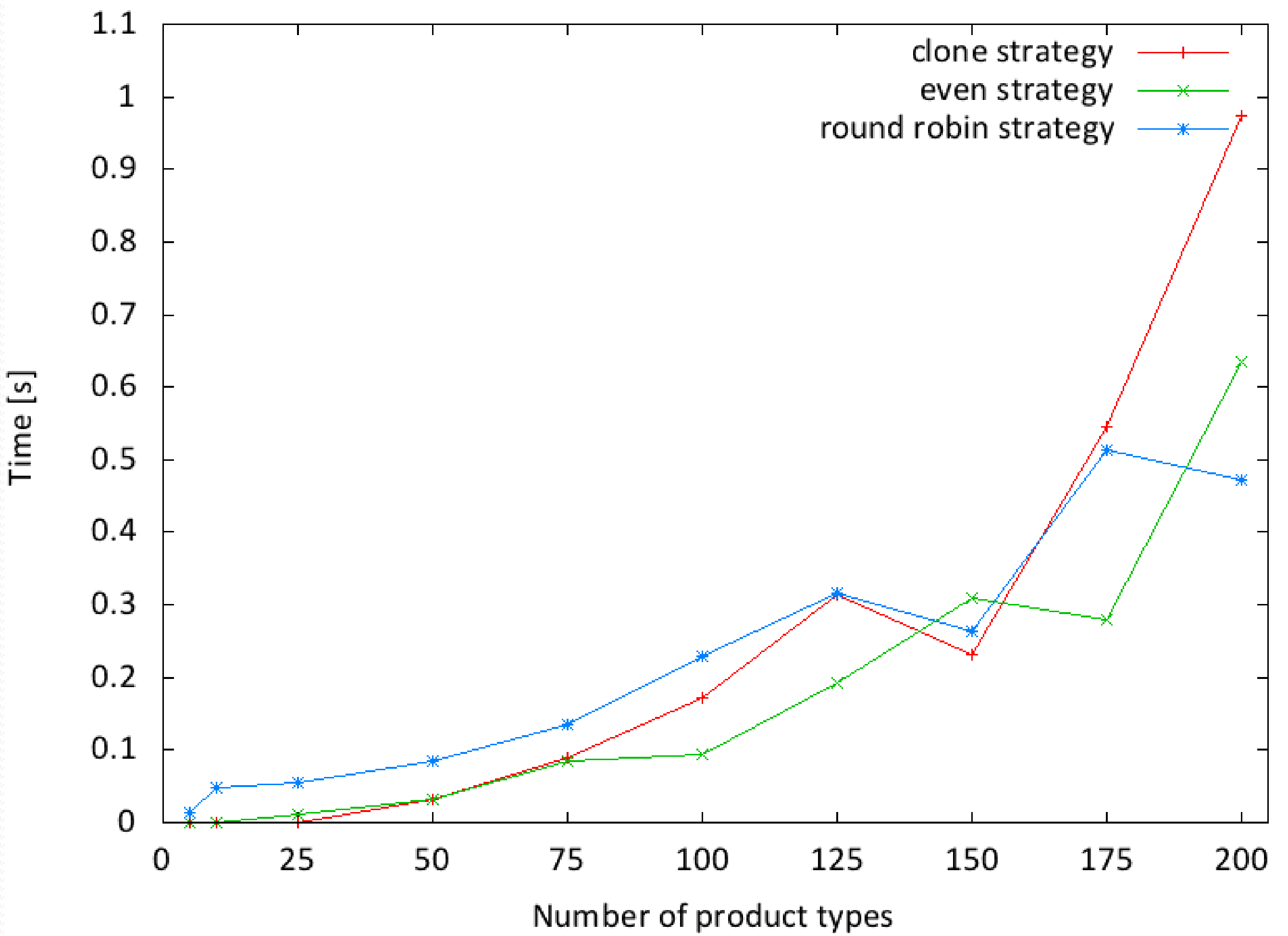
- a single order contains of 5 to 200 product types (from 1 to 10 units of each type)

- prices of products in depots are determined based on
  - basic price
  - modified by discount factor generated with normal distribution [-50%, +50%]

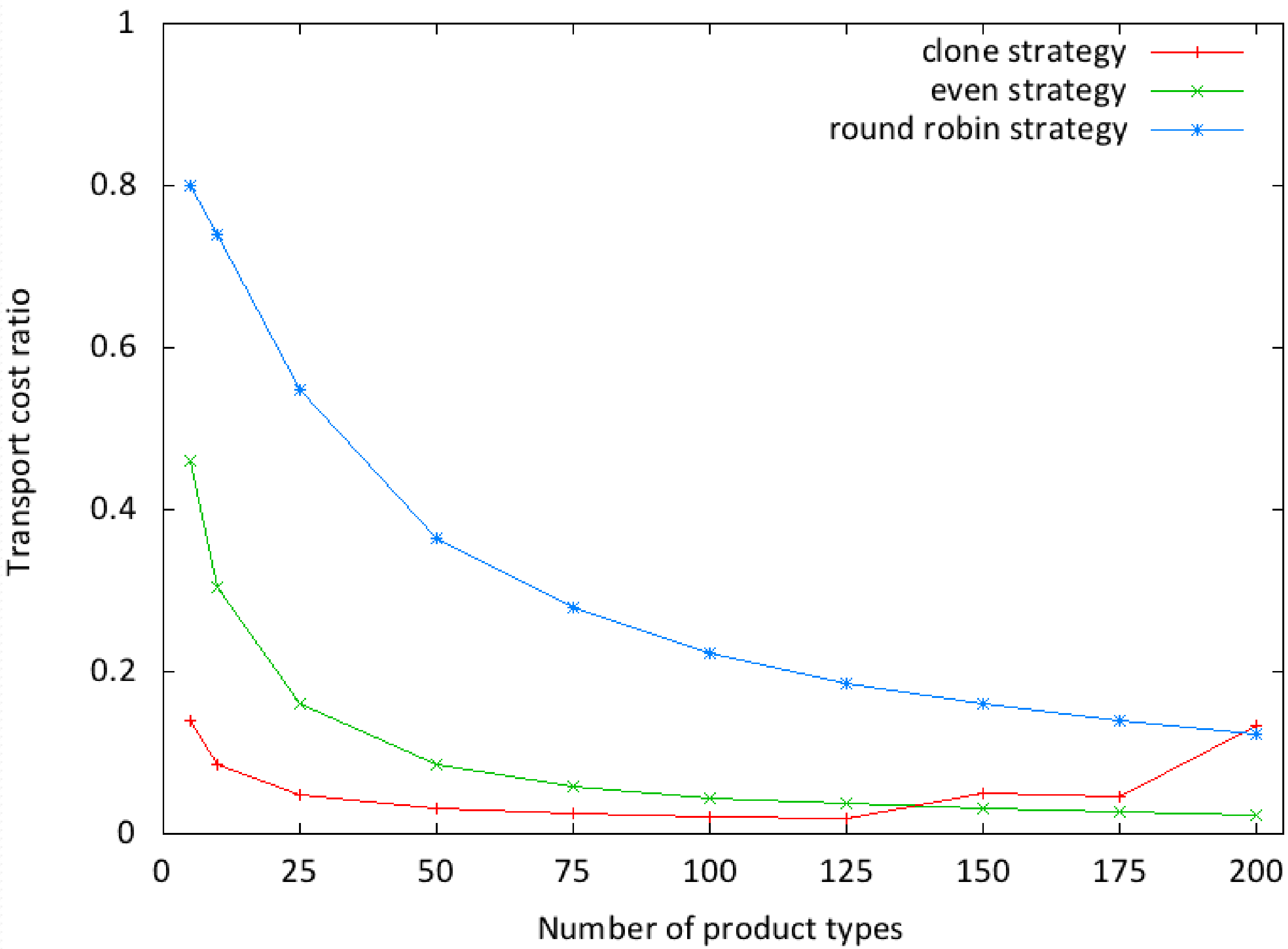
Computational Experiments

- availability of products in depots are generated according to 3 scenarios:
  - „round robin" distribution
    - demanded units of a product are placed in depots one by one
    - all depots have to be visited (order delivery is crucial)
  - „clone" distribution
    - all demanded products are available in all depots
    - prices of products are crucial (order completion is crucial)
  - „even" distribution
    - availability in all depots is increased by one unit at the time until demand is exceeded
    - prices and distances are crucial

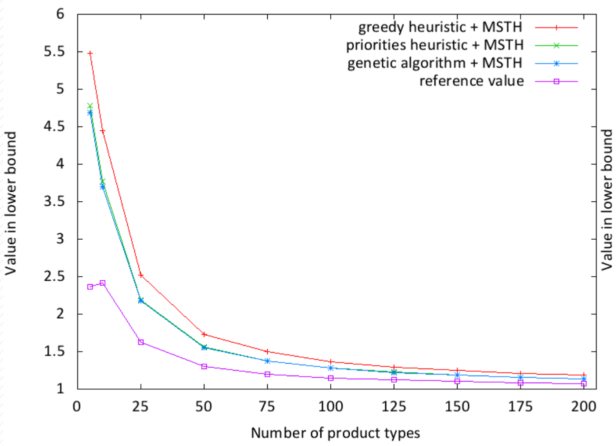
Time per generation



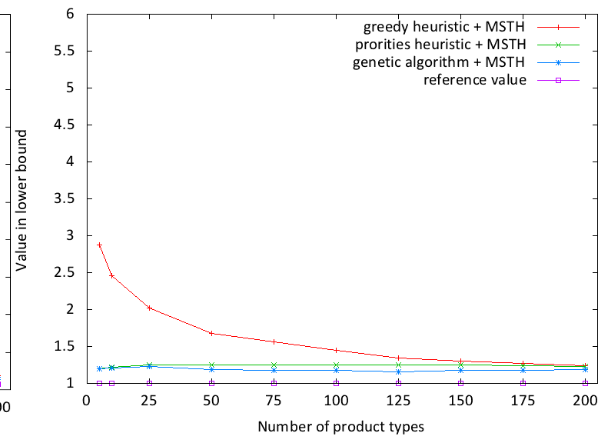
Transportation cost vs. Products cost



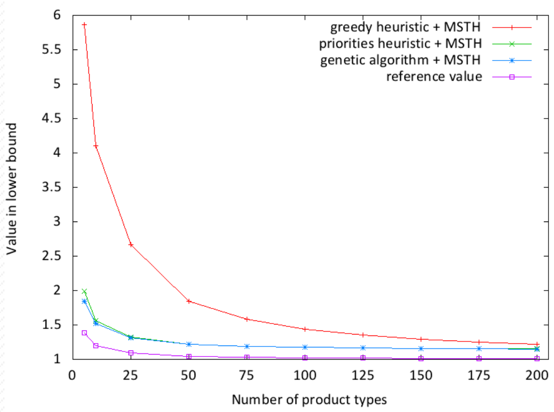
Criterion value vs. lower bound



„Round Robin" distribution



„Clone" distribution



„Even" distribution

Conclusions

- web service devoted for charitable institutions
  - data base of offered/donated products
    and submitted requests
  - optimization tool supporting realizing orders by
    minimizing products cost and transportation cost
- solving a variant of travelling purchaser problem by
  - heuristic list algorithm
  - genetic algorithm
- validation algorithms in computational experiments
  - solution quality close to the lower bound
  - short computational time acceptable by web service users