Scheduling for Electricity Cost in Smart Grid

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Workshop on New Challenges in Scheduling Theory
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Joint work with
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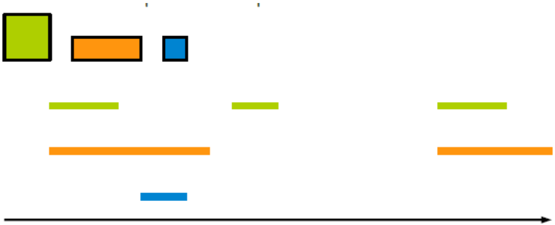
Smart Grid

- Smart Grid improves efficiency and reliability of production of energy
- Consumers can request tasks to operate between time periods for a specified duration
  - E.g.: A washing machine may be requested to operate for 2 hours between 7-11am or 1-5pm



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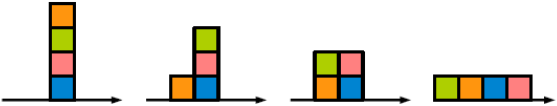
- Energy cost is modelled by a convex function on the load of timeslots [1]
- Objective: Schedule all requests using the minimum energy cost
- Demand response management

[1] Koutsopoulos and Tassiulas 2011

Model

- set $\mathcal{J}$ of n jobs $\{J_1, J_2, \cdots J_n\}$
  unit power requirements, unit duration, arbitrary feasible timeslots
- Cost function: general convex function $f(\ )$



- Time: divided into integral timeslots $T = \{1, 2, 3, \cdots, \tau\}$
- Each job $J_i \in \mathcal{J}$ has feasible timeslots $I_i \subseteq T$, in which J_i can be scheduled
- J_i must be assigned to exactly one feasible timeslot from I_i
- Load $\ell(t)$ of a timeslot t is the # of jobs assigned to it
- Goal: Minimize $\sum_{t \in T} f(\ell(t))$

Results

- Polynomial time optimal offline algorithm [2]
  - $O(n^2\tau)$
  - n jobs, τ # of timeslots
- Improvement in time complexity
  - If the feasible timeslots of a job form contiguous interval
  - from $O(n^2\tau)$ to $O(n\log\tau + \min(n,\tau)n\log n)$

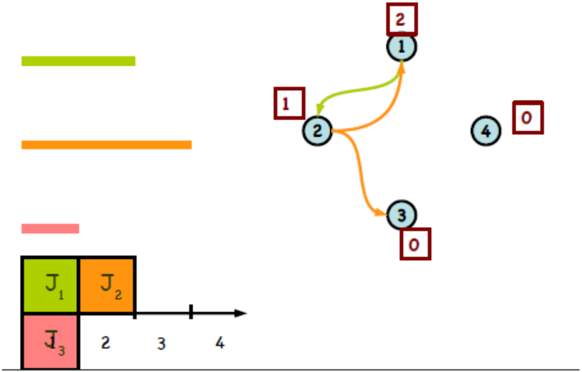
- Note: related to minimum cost maximum flow problem & maximum-cardinality minimum-weight matching on a bipartite graph
  - for more general input, higher time complexity

[2] Burcea, Hon, Liu, W., Yau 2013

Feasible Graph

- Feasible (multi-) graph G: show potential alternative allocation of jobs
- Each timeslot is represented by a vertex
- If J_i is assigned to time slot r, then $\forall w \in I_i \backslash \{r\}$ add an arc (r, w) with J_i as its label

Example: $I_1 = \{1, 2\}$, $I_2 = \{1, 2, 3\}$, $I_3 = \{1\}$



Legal-Path

- A legal-path in G is a path (t, t') such that $\ell(t) - \ell(t') \geq 2$, implying
    - non-optimal assignment
    - possible shift of job assignment to reduce cost

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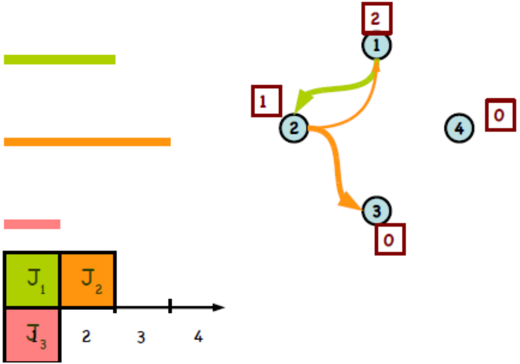

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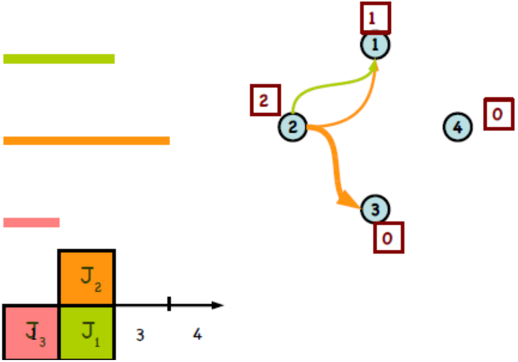

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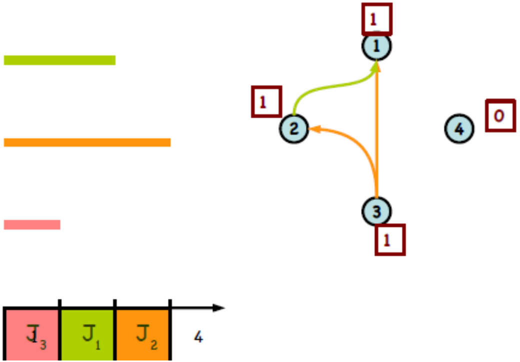


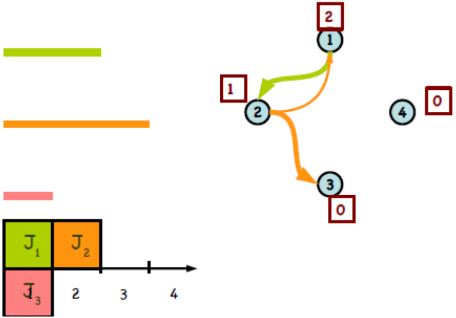
Algorithm

Greedy approach: Order the job arbitrarily, for J_i:

1. assign J_i to a feasible timeslot, say t, with minimum load
2. update feasible graph G
3. if there is a legal-path from t to some t', then shift job assignment along the legal-path

Algorithm — example

- Suppose $\mathcal{J} = \{J_1, J_2, J_3\}$, $T = \{1, 2, 3, 4\}$,
- $I_1 = \{1, 2\}$, $I_2 = \{1, 2, 3\}$, and $I_3 = \{1\}$
  1. J_1: assign it to time 1
  2. J_2: assign it to time 2 (choose between time 2 & 3)
  3. J_3: assign it to time 1
- Legal-path is formed after assigning J_3



Correctness

Theorem

The greedy algorithm finds an optimal solution.

Lemma

If $\exists$ legal-path after adding J_i to timeslot r, then $\exists$ legal-path from r.

Lemma

Suppose there is no legal-path in the feasible graph. Adding a job and shifting jobs along legal-path (if exists) results in no legal-path.

Lemma

Non-existence of legal-path implies optimality of the solution.

Time Complexity

Theorem

We can find the optimal solution in $O(n^2\tau)$-time.

- Feasible graph: τ vertices, at most $n\tau$ arcs
- For each job:
  - update feasible graph upon assigning a job: $O(\tau)$-time
  - BFS to find legal-path: $O(\tau + n\tau)$-time
  - shift and update feasible graph: $O(n\tau)$-time
- Overall: $O(n^2\tau)$-time

Improved Time Complexity - Contiguous Feasible Interval

Theorem

If the feasible timeslots of each job form a contiguous interval, we can find the optimal solution in $O(n \log \tau + \min(n, \tau) n \log n)$-time.

- Comparing with $n^2\tau$:
  - If $n < \tau$, $O(n^2 \log \tau)$
  - If $n > \tau$, $O(n\tau \log n)$

Contiguous Feasible Interval — Ideas

Speed up for

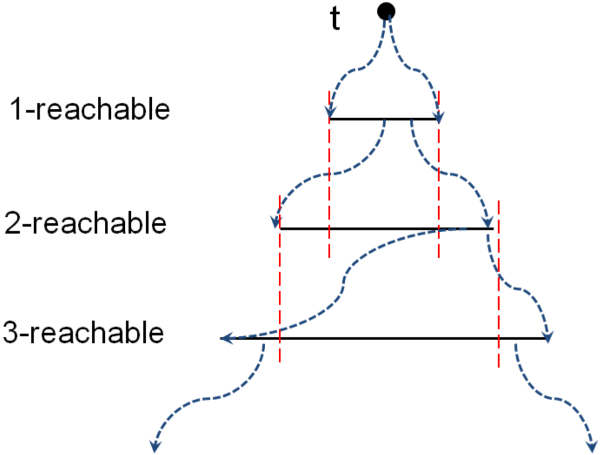
- finding legal path: $O(\min(n,\tau))$-time instead of $O(n\tau)$
- update after shift: $O(\min(n,\tau)\log n)$-time instead of $O(n\tau)$

Legal-path from a vertex t

- ℓ-reachable interval: timeslot t' with length-ℓ path from t to t'
- $\ell \le \min(n,\tau)$
- compute reachable intervals using auxiliary data structures including linked list, heaps, dynamic range minimum query

Legal-path

- key: feasible timeslots are contiguous
- depth $\leq \min(n, \tau)$



Related Work and Future Directions

- Online algorithms?
- General problem with arbitrary power requirement / arbitrary duration is NP-hard
  - extend proof for problem with piecewise linear function [3]
  - Approximation algorithms?
- Multi-objective problem to minimize energy and maximum some utility [4]
- Managing load by changing price of electricity over time [5, 6, 7]

[3] Koutsopoulos and Tassiulas 2011

[4] Salinas, Li and Li 2013

[5] Caron and Kesidis 2010

[6] Maharjan, Zhu and Zhang 2013

[7] Mohsenian-Rad et al 2013

Thank you!